Trigonometry

Trigonometric Ratios. Trigonometric ratios are relationships between the sides of a right-angled triangle.

There are three primary trigonometric ratios: sine $(\sin)$, cosine $(\cos)$, and tangent $(\tan)$.

Expressions of the Six Trigonometric Ratios. For a right-angled triangle with angle θ:

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}, \quad \cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}, \quad \tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

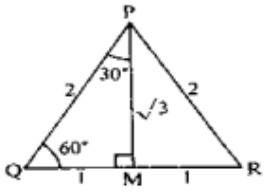
The reciprocals of these ratios are: cosecant $(\csc)$, secant $(\sec)$, and cotangent $(\cot)$:

$$\csc\theta = \frac{1}{\sin\theta}, \quad \sec\theta = \frac{1}{\cos\theta}, \quad \cot\theta = \frac{1}{\tan\theta}$$

Special Angles. Common special angles include: $30°$, $45°$, and $60°$.

30^0 and 60^0

Suppose ΔPQR is equilateral, with sides 2 units and that PM is the perpendicular bisector of QR

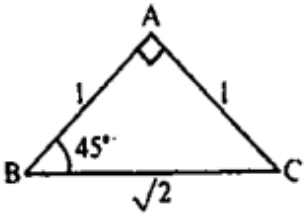


$\sin 30^0 = \frac{1}{2}$; $\cos 30^0 = \frac{\sqrt{3}}{2}$; $\tan 30^0 = \frac{1}{\sqrt{3}} or \frac{\sqrt{3}}{3}$

$\sin 60^0 = \frac{\sqrt{3}}{2}$; $\cos 60^0 = \frac{1}{2}$; $\tan 60^0 = \frac{\sqrt{3}}{1} = \sqrt{3}$

45^0

Consider a right-angled triangle which is isosceles and in which the equal sides are 1 unit in length. The equal angles will each be 45^0



$\sin 45^0 = \frac{1}{\sqrt{2}} or \frac{\sqrt{2}}{2}$; $\cos 45^0 = \frac{\sqrt{2}}{2}$; $\tan 45^0 = \frac{1}{1} = 1$

These results are summarised in the table below and should be memorised for future use.

Angle	sin	cos	tan
0°	0	1	0
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
45°	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
90°	1	0	∞

Example 1

Show that $cos^2 30^0 + \cos 60^0 \sin 30^0 = 1$

Solution

The left hand side is $cos^2 30^0 + \cos 60^0 \sin 30^0$

$$= \cos 30^0 (\cos 30^0) + \cos 60^0 \sin 30^0$$

$$= \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + \frac{1}{2} \times \frac{1}{2} = \frac{3}{4} + \frac{1}{4}$$

$$= 1 \text{ as required}$$

TYPES OF ANGLES

A CUTE: Between 0 and 90

OTBUSE: Greater than 90 but less than 180

REFLEX: Greater than 180 but less than 360

Graphs of Trigonometric Functions. The graphs of $\sin\theta$ and $\cos\theta$ are periodic, oscillating between -1 and

Complete the un answered parts of the example below;

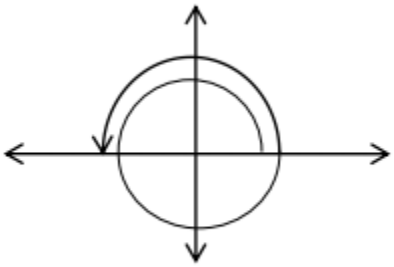
Example I

Write down the values of the following, leaving surds in your answers (*the calculator should not be used*).

(a) cos 780°
(b) sin 780°
(c) tan 780°
(d) sin 540°
(e) cos 540°
(f) cos 210°
(g) sin 150°
(h) sin(-270°)
(i) sin 225°
(j) sin 405°
(k) tan(-60°)

SOLUTION

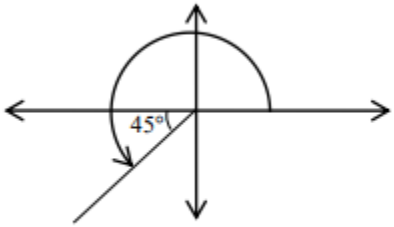
sin 540°



$\sin 540° = \sin 180° = 0°$

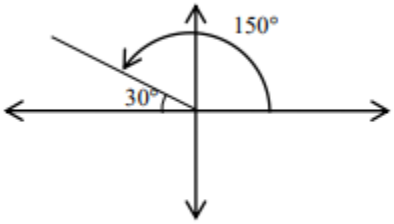
$\cos 540° = \cos 180° = 0°$

sin 225°



$$\sin 225° = -\sin 45° = \frac{-1}{\sqrt{2}}$$

sin 150°



$$\sin 150 = +\sin 30 = \frac{1}{2}$$