

GREENHILL ACADEMY SECONDARY
S.5 MATHS HOLIDAY WORK TERM III 2024

PAPER 1

1. Differentiate: $\log_e(1 - 2x^2)^{-\frac{1}{2}}$ with respect to x .
2. Find $\int \frac{x^4 - x^3 + x^2 + 1}{x^3 + x} dx$
3. Express the function $f(x) = \frac{3x^3 + 2x^2 - 3x - 1}{x(x^2 - 1)}$, as the sum of partial fractions and hence find $\int_1^2 f(x) dx$
4. Hence find the greatest and least values of $\frac{1}{8 \sin x - 6 \cos x - 7}$.
5. A man invests £ 2,000 at the beginning of each year and receives compound interest at 5% per annum. By forming a series, show that the total amount of accumulated capital and interest at the end of n years is £ $42000(1.05^n - 1)$. Determine also the least number of years that the man would need to continue his investments in order to accumulate not less than £ 100,000.
6. Using the substitution $x = 2 \cos \theta$, deduce that $\int_1^2 \frac{1}{x^2 \sqrt{4 - x^2}} dx = \frac{\sqrt{3}}{4}$.
7. Differentiate $\log_5 \left(\frac{e^{\sin x}}{\cos^2 x} \right)$ with respect to x
8. Given that $f(x) = 4x^2 - 8x + 13$ Express $f(x)$ in the form $+b(x + c)^2$. Hence; find the minimum value of $f(x)$, stating the value of x for which it occurs. Evaluate $\int_0^1 \frac{1}{f(x)} dx$.

PAPER 2:

Attempt All questions

1. A set digits consists of x zeros and n ones.
(a) Find the mean of this set of data.
(b) Hence show that the variance is $\frac{xy}{(x+y)^2}$. (05 marks)
2. The resultant of forces $F_1 = 3\hat{i} + (a - c)\hat{j}$, $F_2 = (2a + 3c)\hat{i} + s\hat{j}$ and $F_3 = 4\hat{i} + 6\hat{j}$ acting on a particle is $10\hat{i} + 12\hat{j}$. Find;
(i) The value of a and c .

(ii) Magnitude of force F_2 (05 marks)

3. Calculate a weighted price index for the following figures for 1994 based on 1980. Hence comment on your results. (05 marks)

Item	1980 (shs)	Weight	1994	Weight
Food	50	3	60	4
Housing	40	2	50	3
Transport	16	1	20	2

4. A car, A travelling at a constant velocity of 25ms^{-1} , over takes a stationary car B. Two seconds later, car B sets off in pursuit, accelerating at 6ms^{-2} . How far does B travel before catching up with A? (05 marks)

5. Two events M and N are such that $P(M) = 0.7$, $P(M \cap N) = 0.45$ and $P(M^1 \cap N^1) = 0.18$. Find;

(a) $P(N^1)$

(b) $P(M \text{ or } N \text{ but no Both } M \text{ and } N)$ (05 marks)

6. Given that $y = \sin\theta$ is measured with a maximum possible error of 2%. If $\theta = 30^\circ$, determine the;

(i) Absolute error in y.

(ii) Interval within which the values of y lies. Give your answer correct to 4 significant figures. (05 marks)

7. The engine of a lorry of mass 5000kg is working at steady rate of 350kw against a constant resistance force of 1000N. the lorry ascends a slope of inclination θ° to the horizontal. If the maximum speed of the lorry is 20ms^{-1} , find the value of θ . (05 marks)

8. A weighing scale is corrupted such that the actual weights 24g and 54g are recorded as 35g and 60g respectively. Use linear interpolation/extrapolation to estimate;

(i) The weight which is recorded accurately.

(ii) The weight which the scale reads when it is unloaded.

(05 marks)

SECTION B

9. A random variable X has the probability distribution function given by;

$$P(X = x) = \begin{cases} \frac{c}{17} [2^{9-x}]; & x = 0, 1, 2, \dots, n \\ 0; & \text{else where} \end{cases}$$

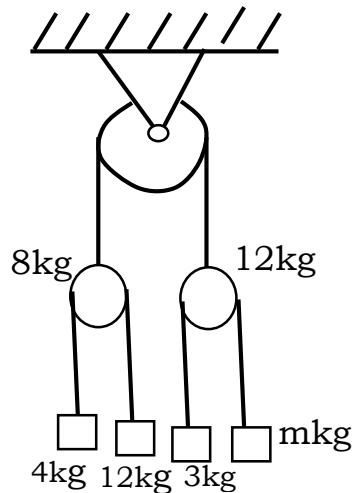
- (i) Find the value of n and c given that $P(X = n) = \frac{1}{255}$
 (ii) Find $P(X > 2 / X \neq 2)$.
 (12marks)

10. (a) Use the trapezium rule with **six** ordinates to find the approximate value of $\int_2^5 x e^{-x} dx$ correct to 3 significant figures.

(b) Find the area bounded by the curve $y = x e^{-x}$ between $x = 2$ and $x = 5$.

(c) Find the percentage error in (a) and b above.
 (12marks)

11. The diagram below shows two pulleys of masses 8kg and 12kg connected by a light inextensible string hanging over fixed pulley



The acceleration of 4kg and 12kg masses are $\frac{g}{2}$ upwards and $\frac{g}{2}$ down wards respectively. The acceleration of the 3kg and mkg masses are $\frac{g}{3}$ upwards and $\frac{g}{3}$ down wards respectively. The hanging portions of the strings are vertical. Given that the string of the fixed pulley remains stationary, find the;

- (a) Tensions in the strings. (09marks)
 (b) Value of m. (03marks)

12. Given that equation $3x^2 + x - 5 = 0$.

(a) (i) Show that the equation has a root between $x = 1$ and $x = 1.5$.

(ii) Hence use linear interpolation to obtain an approximation of the root.

(b) Use Newton Raphson's formula to find the root of the equation by performing two relations, correct to two decimal places.

13. The table below shows height in centimeters of 25 students in a certain school.

Height (cm)	<10	<20	<25	<30	<50	<55	<65
Number of students	0	3	7	15	17	23	25

(a) Calculate the mean height.

(b) Draw an ogive and use it to obtain the middle 70% of the height.

14. A ship P steaming at 20kmh^{-1} in the direction $N50^\circ E$ is 120km due west of a ship Q steaming at 12kmh^{-1} in the direction $N30^\circ W$. find the;

(a) Shortest distance between them in the subsequent motion and the time taken to reach this position.

(b) The time for which the ships are within a range of 50km from each other.

15. (a) Two tetrahedral dice each with four faces numbered 1 to 4 inclusive are tossed at once. If the random variable X represents the sum of the scores on the faces on which the dice lands on.

(i) Generate the probability distribution for X.

(ii) Sketch the p.d.f $f(x)$ and determine the expected value.

Mathematics is the language of the universe, explore it's beauty on holiday

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